Many-user multiple-access channels with random user activity and coding

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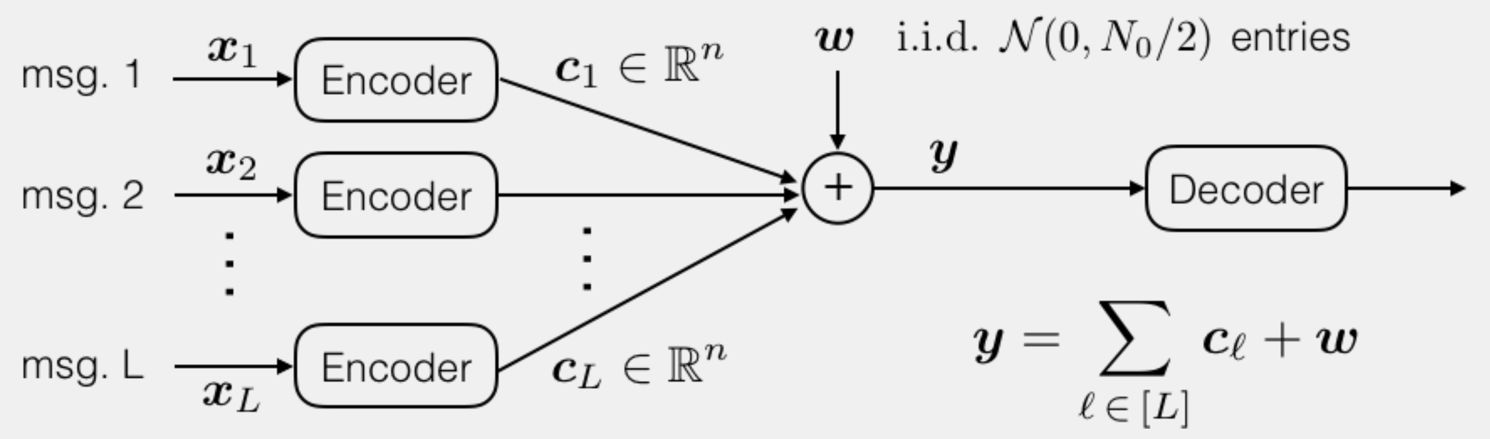
Gaussian multiple access channel (GMAC)

Many-user setting [Pol17, ZPT19]:



- Massive number L of users
- Small fixed data payload for each user: $k = \log_2 M$ bits/user
- Energy-per-bit constraint $\|c_\ell\|^2/k \le E_b$ for constant E_b, $\ell \in [L]$
- Per-user probability of error (PUPE) $:= \frac{1}{L}\sum_{\ell\in[L]} \mathbb{P}(\hat{x}_\ell \neq x_\ell)$

Our setting:

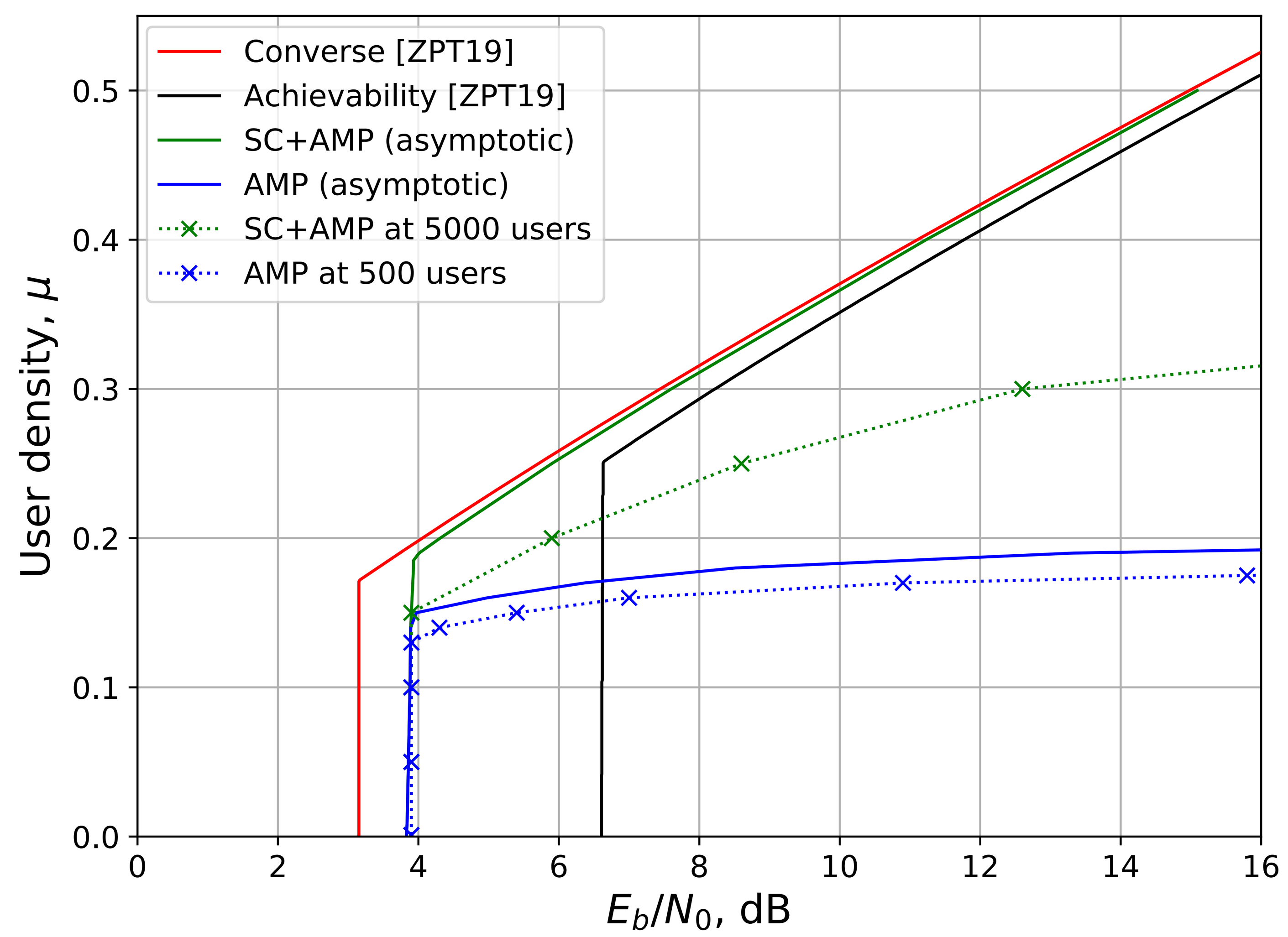
- Each user has its separate codebook
- Code length $n \to \infty$ as $L \to \infty$ with user density $\mu = L/n$ fixed

Existing literature

Q: Given μ, what is minimum E_b/N_0 required for target PUPE?

Coding strategies:

- Random Gaussian codebooks + Maximum-Likelihood (ML) decoding (theoretical, infeasible) [Pol17, ZPT19]
- Random Gaussian codebooks (with band-diagonal structure) + Approximate Message Passing (AMP) decoding (practical) [HRV22]



GMAC with random user activity

- Each user independently inactive with probability α
- Errors: Misdetections (MD), False Alarms (FA), Active-user Errors (AUE)

Q: Given μ, what is minimum E_b/N_0 required for target error rates?

New achievability bound

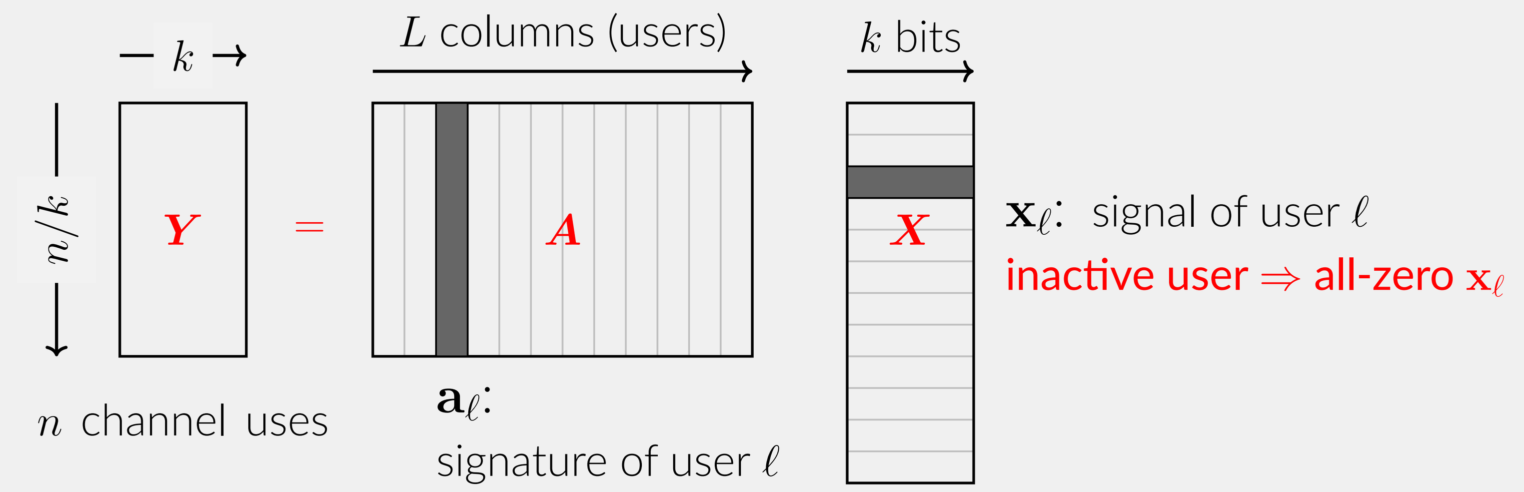
Proof sketch: (following [Pol17, NLDGiA23])

- Generate codebook $\{c_1^{(\ell)}, c_2^{(\ell)}, \dots, c_M^{(\ell)} \overset{\text{iid}}{\sim} \mathcal{N}_n(\mathbf{0}, P\mathbf{I}_n)\}$ for each user
- Let $w_\ell \in \{\emptyset, 1, 2, \dots, M\}$ index codeword of user ℓ, with $\emptyset$ = inactive user
- Transmitted set: $\mathcal{W} := \{(\ell, w_\ell) : w_\ell \neq \emptyset\}$ with $K_a := |\mathcal{W}|$
- Decoder sees: $y = \sum_{j:(j,w_j)\in\mathcal{W}} c_{w_j}^{(j)}$ + noise
- ML-decoder: returns decoded set $\widehat{\mathcal{W}}$ with $\widehat{K}_a := |\widehat{\mathcal{W}}| < L$; $\widehat{\mathcal{W}}$ contains 1 or 0 codeword from each codebook.
- Error analysis: quantify *unavoidable* errors due to $K_a \neq \widehat{K}_a$ + carefully categorise *additional* errors

$$p_{\mathrm{MD}} := \mathbb{E}\left[\frac{1}{K_a}\sum_{j:(j,w_j)\in\mathcal{W}} \mathbb{1}\{\widehat{w}_j = \emptyset\}\right], \quad p_{\mathrm{AUE}} := \mathbb{E}\left[\frac{1}{K_a}\sum_{j:(j,w_j)\in\mathcal{W}} \mathbb{1}\{\widehat{w}_j \neq w_j\}\right]$$

p_{FA} : analogous

Random linear coding + AMP decoding

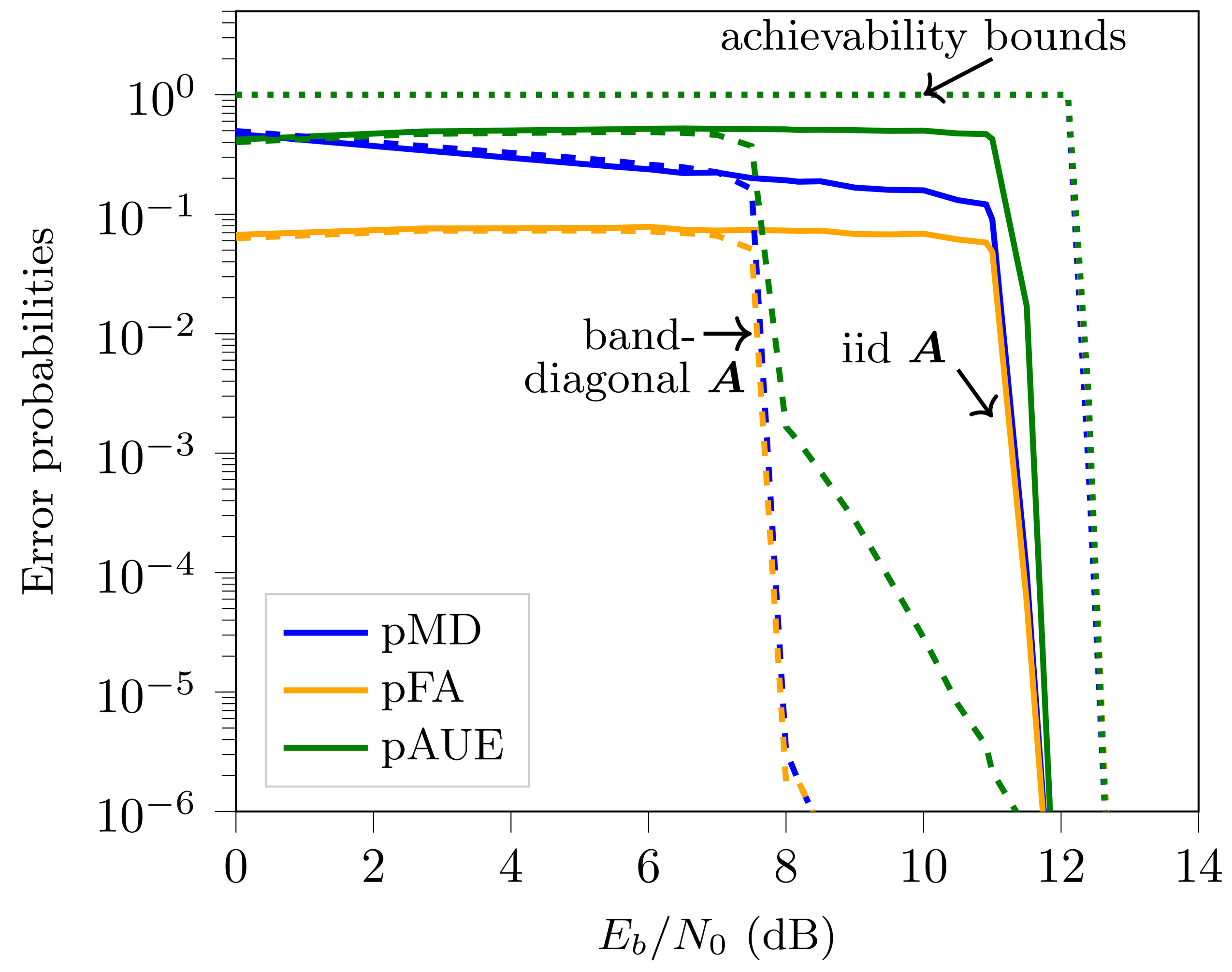


- A : Gaussian matrix (either iid or with band-diagonal structure)
- Decoder sees: $Y = AX$ + noise
- AMP decoder: Initialise with $X^0 = \mathbf{0}$, and for $t \ge 0$,

$$Z^t = Y - AX^t + \frac{1}{n} Z^{t-1}\left[\sum_{j=1}^{L} \eta_{t-1}'\left(\left(A^\top Z^{t-1} + X^{t-1}\right)_j\right)\right]^\top$$

$$X^{t+1} = \eta_t\left(A^\top Z^t + X^t\right).$$

- Key property: $A^\top Z^t + X^t$ distributed row-wise iid as $\bar{X} + G_t \in \mathbb{R}^k$.
  $\bar{X}$: prior, G_t: Gaussian w. zero-mean & characterisable covariance
- Optimal denoiser: $\eta_t(S)$ with ℓ-th row $= \mathbb{E}[\bar{X} \mid \bar{X} + G_t = s_\ell]$

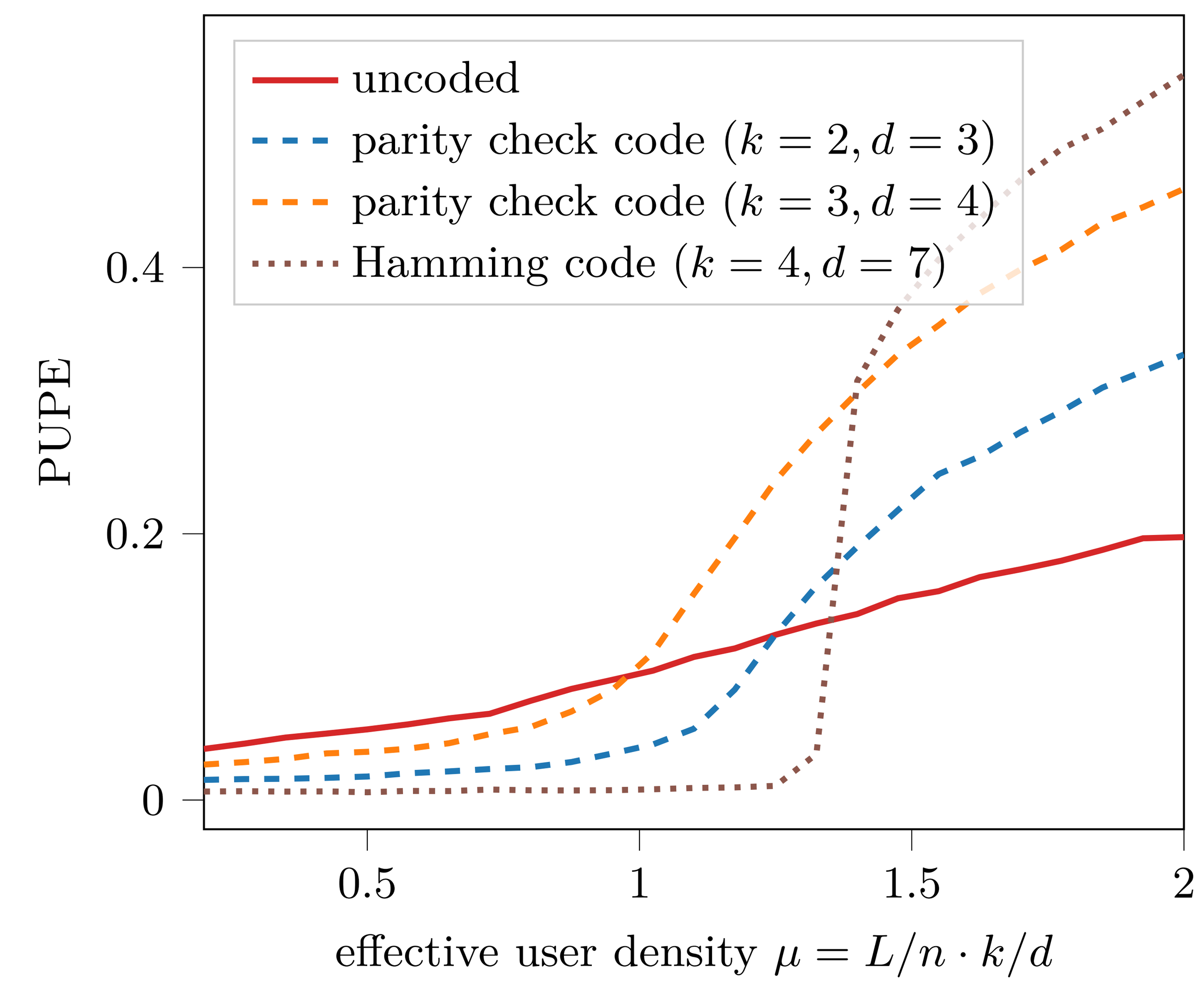


GMAC with coding

- User data is encoded with rate k/d (msg bits/coded bit)
- Effective user density $\mu := L/n \cdot k/d$

Q: Given μ, E_b/N_0, how much reduction in PUPE can coding bring?

- AMP decoder: can flexibly tailor to codeword prior



Ongoing and future work

- LDPC-coded signal and AMP + belief propagation (BP) decoder
- Extension to unsourced random access

[HRV22] Kuan Hsieh, Cynthia Rush, and Ramji Venkataramanan. Near-optimal coding for many-user multiple access channels. *IEEE Journal on Selected Areas in Information Theory*, 3(1):21–36, 2022.

[NLDGiA23] Khac-Hoang Ngo, Alejandro Lancho, Giuseppe Durisi, and Alexandre Graell i Amat. Unsourced multiple access with random user activity. *IEEE Transactions on Information Theory*, 69(7):4537–4558, 2023.

[Pol17] Yury Polyanskiy. A perspective on massive random-access. In *2017 IEEE International Symposium on Information Theory (ISIT)*, pages 2523–2527, 2017.

[ZPT19] Ilias Zadik, Yury Polyanskiy, and Christos Thrampoulidis. Improved bounds on gaussian mac and sparse regression via gaussian inequalities. In *2019 IEEE International Symposium on Information Theory (ISIT)*, pages 430–434, 2019.