Inferring change points in high-dimensional linear regression via Approximate Message Passing

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Joint work with
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Stock prices

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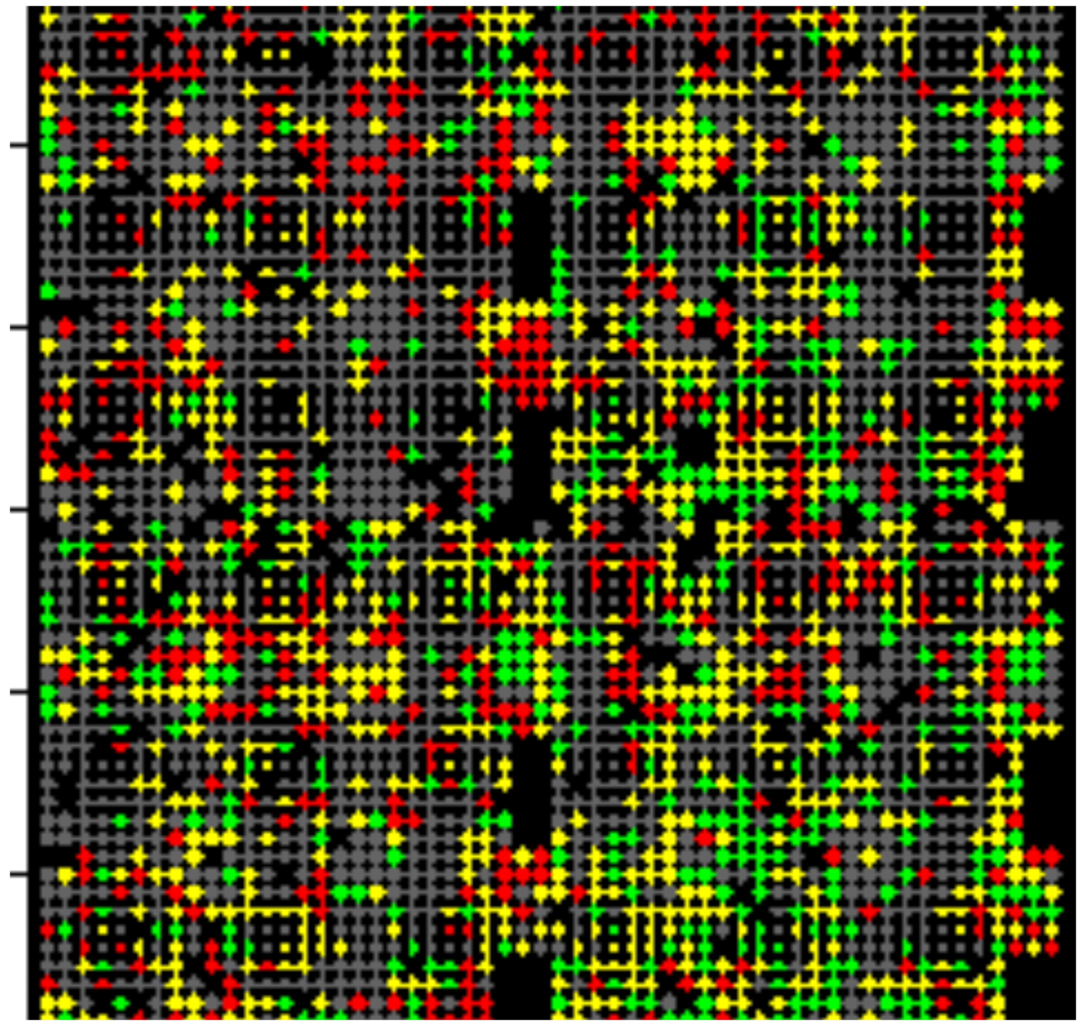

Gene expression

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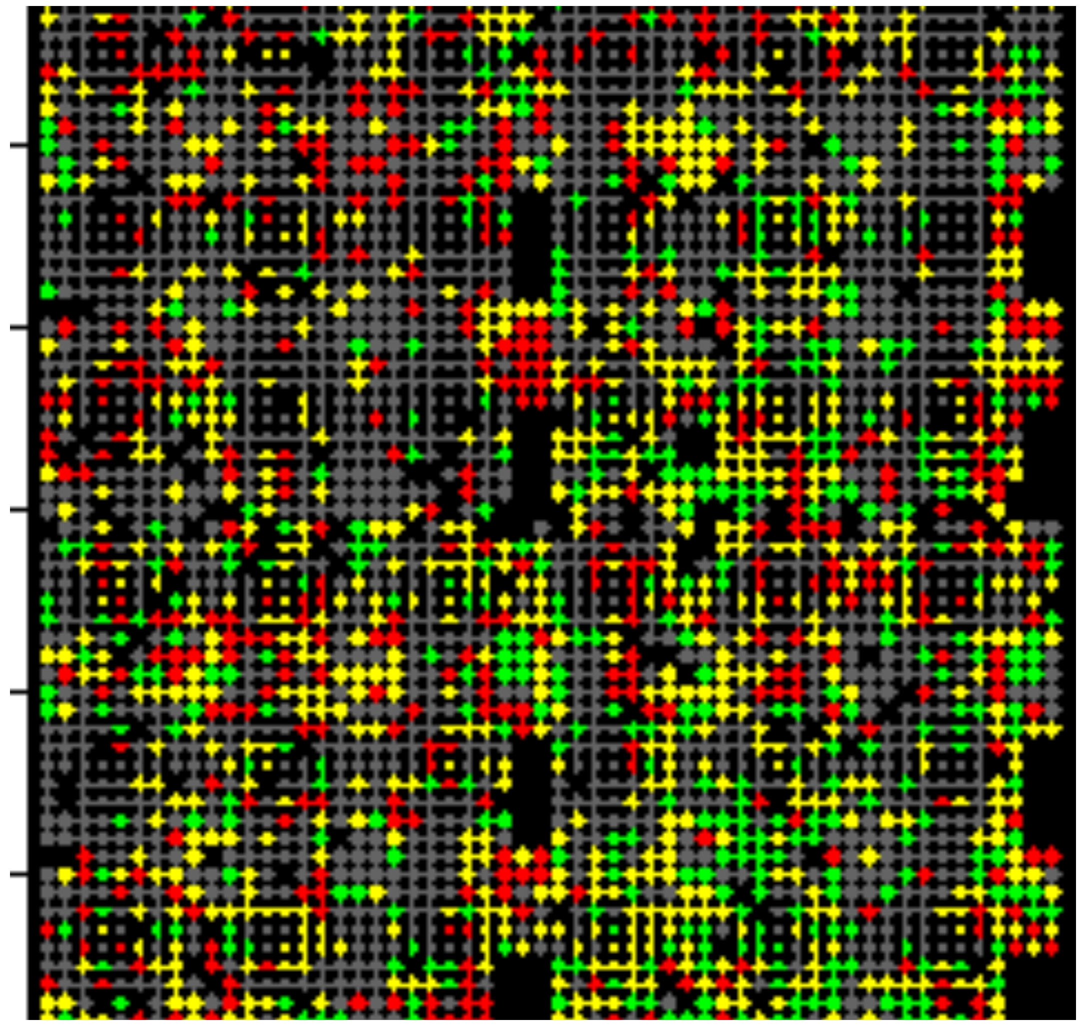

Gene expression

Want to understand how underlying **data generating mechanisms** change over time

For $i = 1, 2, \ldots, n$,

$$\boldsymbol{x}_i, \boldsymbol{\beta} \in \mathbb{R}^p$$

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$$y_i = \boldsymbol{x}_i^\top \boldsymbol{\beta} + \varepsilon_i$$

For $i = 1, 2, \ldots, n$,

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Goal: recover change point locations $\{\eta_l\}_{l=1}^{L^*-1}$ from $\{\boldsymbol{x}_i, y_i\}_{i=1}^{n}$, and estimate signals $\{\boldsymbol{\beta}^{(l)}\}_{l=1}^{L^*}$

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Scaling regime: $n, p \to \infty$, $n/p \to$ fixed constant δ

$$\hat{\eta}_1, \ldots, \hat{\eta}_{L^*-1} = \underbrace{\arg\min_{\tilde{\eta}_1 \leq \tilde{\eta}_2 \leq \ldots \leq \tilde{\eta}_{L^*-1}} \sum_{l=1}^{L^*}}_{\text{search through different partitioning}} \underbrace{\min_{\tilde{\boldsymbol{\beta}} \in \mathbb{R}^p} \sum_{i=\tilde{\eta}_{l-1}+1}^{\tilde{\eta}_l} \left(y_i - \boldsymbol{x}_i^\top \tilde{\boldsymbol{\beta}}\right)^2}_{\text{Residual Sum of Squares}}$$

$p \ll n$

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~~Residual Sum of Squares~~ ~~$p \ll n$~~

penalised maximum likelihood e.g., LASSO

$n, p \to \infty$

[Zhang et al. 2015, Leonardi and Bühlmann 2016, Lee et al. 2016, Rinaldo et al. 2021]

Sparse prior: penalised maximum likelihood + partitioning

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Limitations:

- Restricted to certain signal priors
- Provide point estimates, without uncertainty quantification

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We address these limitations via:

Approximate Message Passing (AMP)

[Donoho et al. 2009, Bayati and Montanari 2011, Rangan 2011, Berthier et al. 2019, Celentano and Montanari 2022, Feng et al. 2022, Gerbelot and Berthier 2023]

Aside: AMP for sparse recovery

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$$\boxed{\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad \begin{array}{l} \text{entries of } \boldsymbol{\beta} \overset{\text{iid}}{\sim} p_{\bar{\beta}} \\ \text{entries of } \boldsymbol{X} \overset{\text{iid}}{\sim} N(0,1/n) \end{array}}$$

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entries of $\boldsymbol{\beta} \overset{\text{iid}}{\sim} p_{\bar{\beta}}$

entries of $\boldsymbol{X} \overset{\text{iid}}{\sim} \mathcal{N}(0,1/n)$

modified residual: $\boldsymbol{r}^t = \boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}^t + b_t \boldsymbol{r}^{t-1}$

signal estimate: $\boldsymbol{\beta}^{t+1} = \eta_{\substack{\text{soft}\\\text{thres}}}\left(\boldsymbol{\beta}^t + \boldsymbol{X}^\top \boldsymbol{r}^t; \theta_t\right)$

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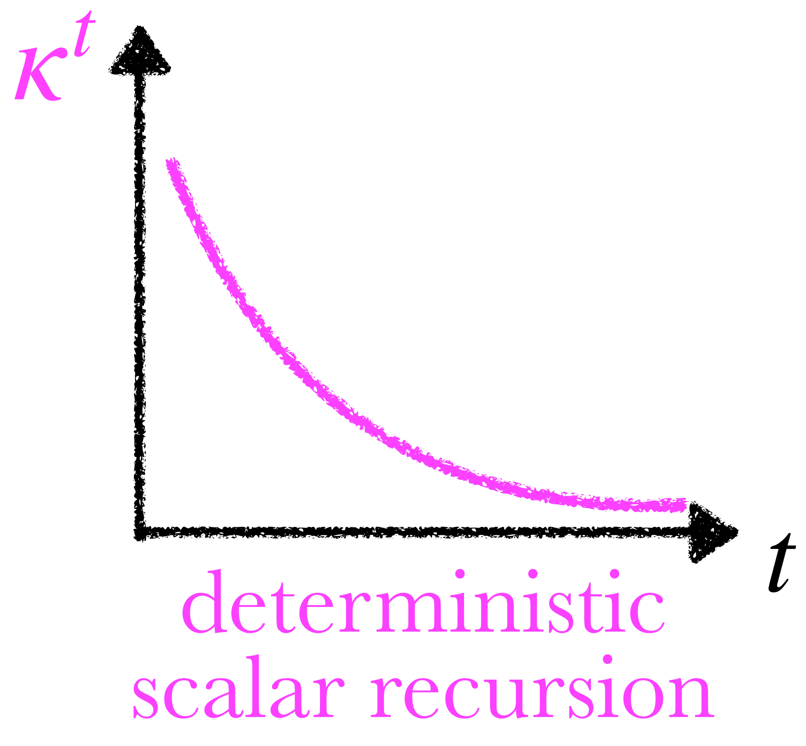

AMP for change point inference

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Signal is $\boldsymbol{B} = \begin{bmatrix} \uparrow & & \uparrow \\ \boldsymbol{\beta}^{(1)} & \dots & \boldsymbol{\beta}^{(L)} \\ \downarrow & & \downarrow \end{bmatrix} \sim p_{\bar{\boldsymbol{B}}}$ and define $\boldsymbol{\Theta} := \boldsymbol{X}\boldsymbol{B}$.

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$$\boldsymbol{R}^t = g^t\left(\boldsymbol{\Theta}^t, \boldsymbol{y}\right)$$

produce residual $\boldsymbol{R}^t$

infer change points

$$\boldsymbol{B}^{t+1} = X^\top \boldsymbol{R}^t - \hat{\boldsymbol{B}}^t\left(\boldsymbol{C}^t\right)^\top$$

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$$f^t\left(\boldsymbol{B}^t\right) = \mathbb{E}\left[\bar{\boldsymbol{B}} \mid \bar{\boldsymbol{B}} + \boldsymbol{G}_B^t = \boldsymbol{B}^t\right]$$

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Theorem (informal)

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$$\boldsymbol{B}^t \overset{\mathbb{P}}{\simeq} \boldsymbol{B} + \boldsymbol{G}_{\boldsymbol{B}}^t \qquad \text{rows of } \boldsymbol{G}_{\boldsymbol{B}}^t \overset{\text{iid}}{\sim} N(\boldsymbol{0}, \boldsymbol{\kappa}_{\boldsymbol{B}}^t)$$

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Proposition (informal) (Hausdorff distance d_H)

$$d_H\left(\boldsymbol{\eta}, \hat{\boldsymbol{\eta}}(\boldsymbol{\Theta}^t, \boldsymbol{y})\right)/n \overset{\mathbb{P}}{\simeq} \mathbb{E}_{\boldsymbol{V}^t_{\boldsymbol{\Theta}}, \bar{\boldsymbol{\Theta}}}\left[d_H\left(\boldsymbol{\eta}, \hat{\boldsymbol{\eta}}(\bar{\boldsymbol{\Theta}}\boldsymbol{\nu}^t_{\boldsymbol{\Theta}} + \boldsymbol{G}^t_{\boldsymbol{\Theta}}, \bar{\boldsymbol{y}}(\bar{\boldsymbol{\Theta}}, \boldsymbol{\eta}))\right)/n\right]$$

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Similar result in terms of posterior $p(\boldsymbol{\eta} \,|\, \boldsymbol{\Theta}^t, \boldsymbol{y})$

Simulation results

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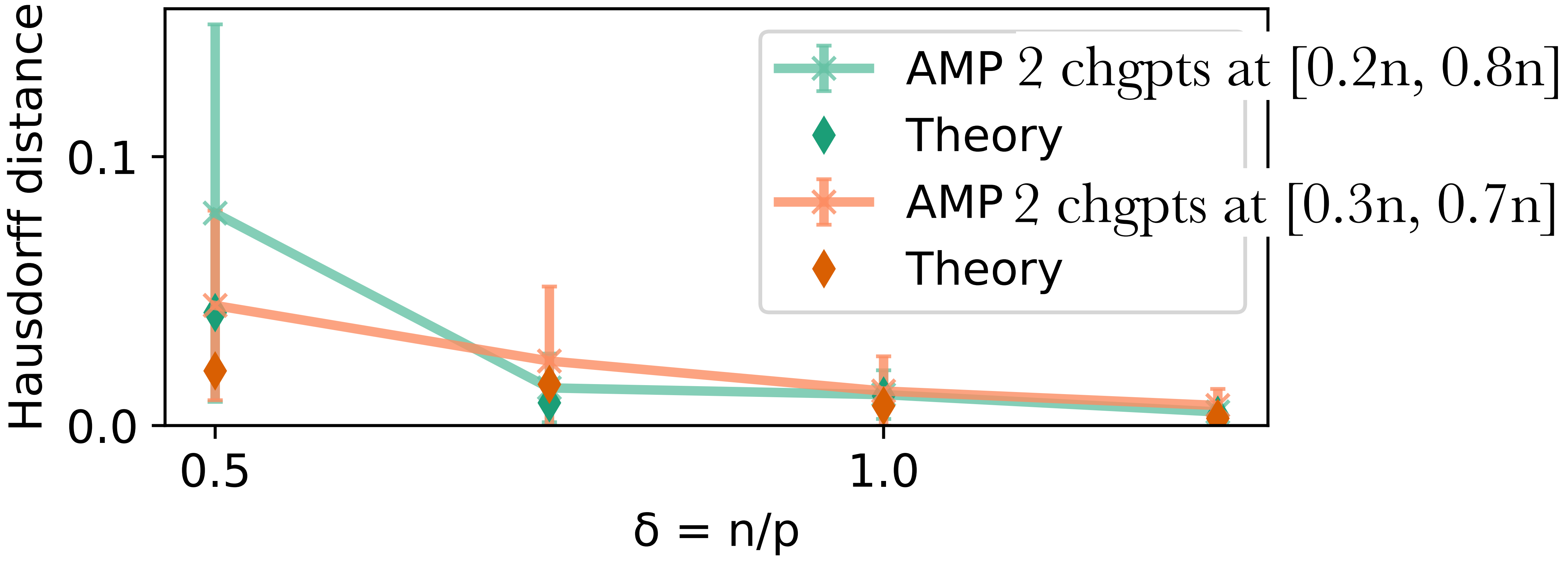
Hausdorff distance



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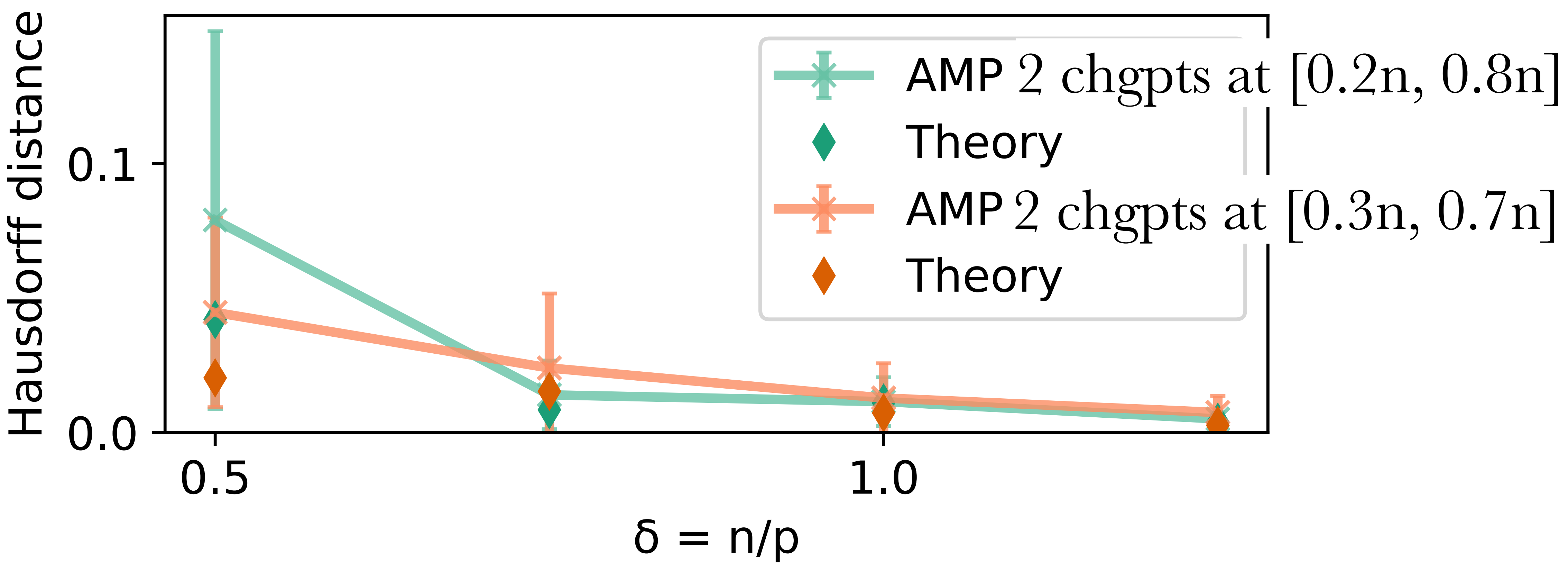
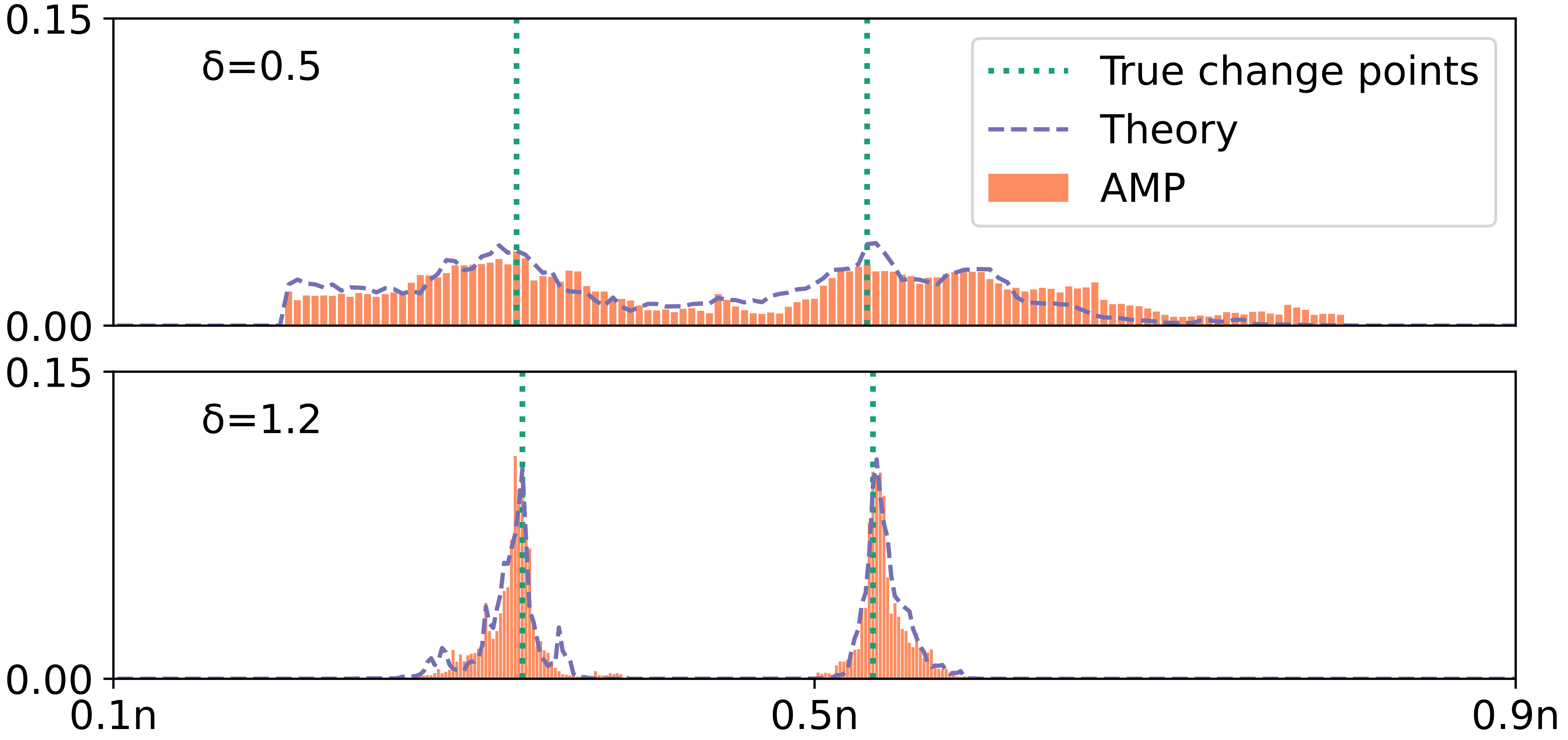


Posterior



Comparison with other algorithms

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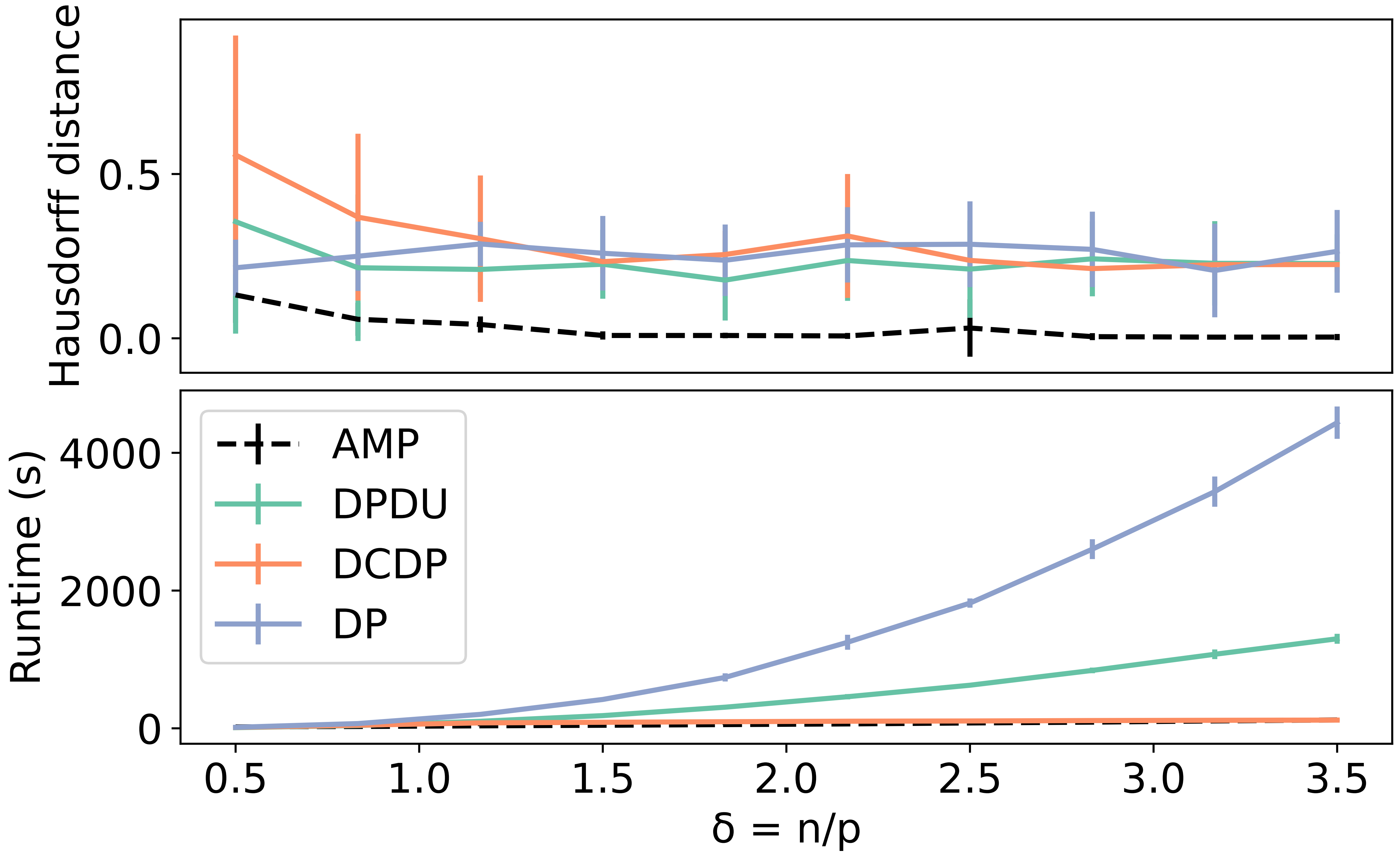

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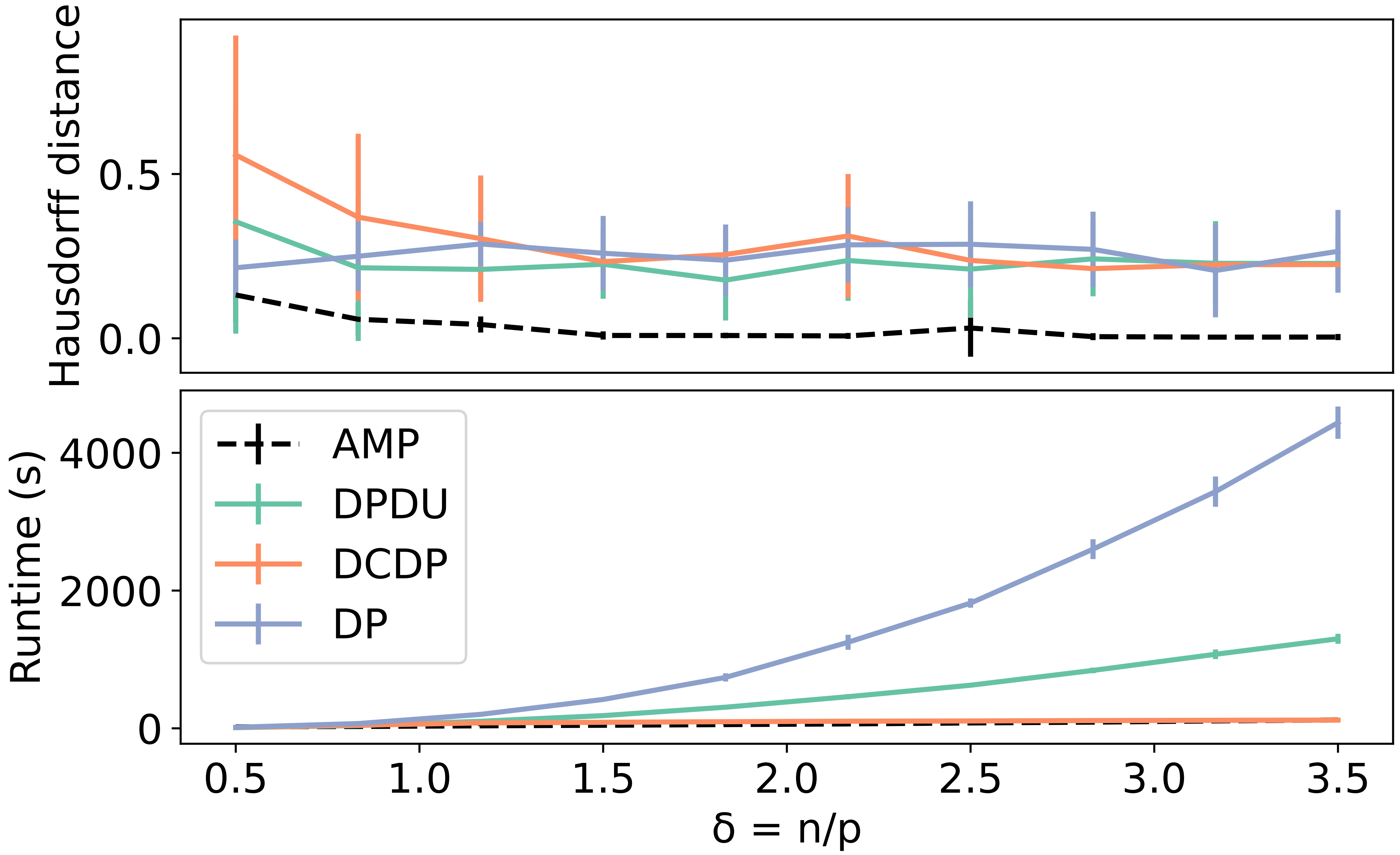


Computational cost: LASSO+partitioning: $O(n^2p^2)$, $O(n^3)$

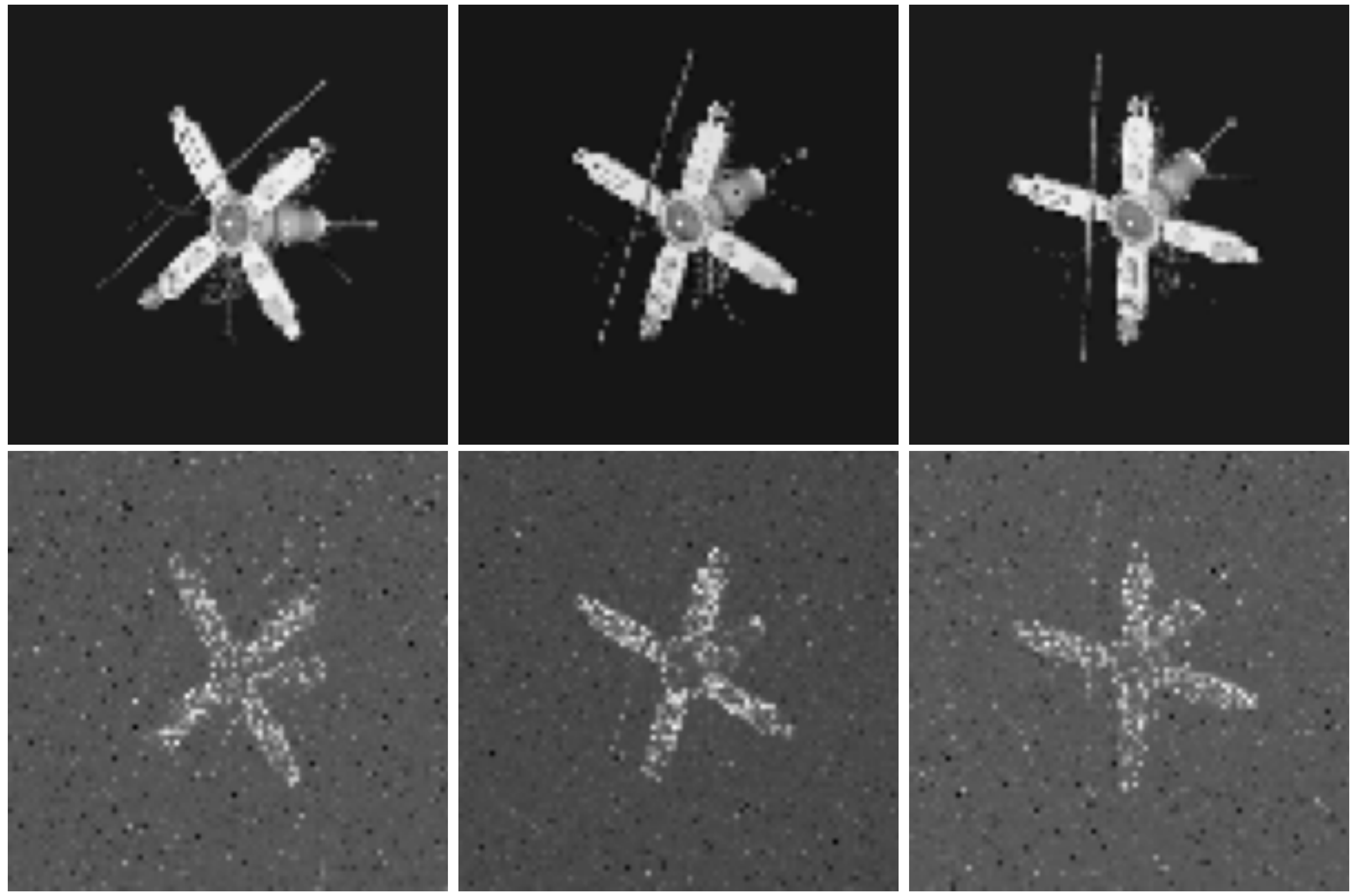
AMP: $O(np)$

Experiments on real images

$\boldsymbol{\beta}^{(1)}$	$\boldsymbol{\beta}^{(2)}$	$\boldsymbol{\beta}^{(3)}$
Rotated 0⁰	Rotated 30⁰	Rotated 45⁰

Ground truth signals:

Estimated signals after 10 AMP iterations:



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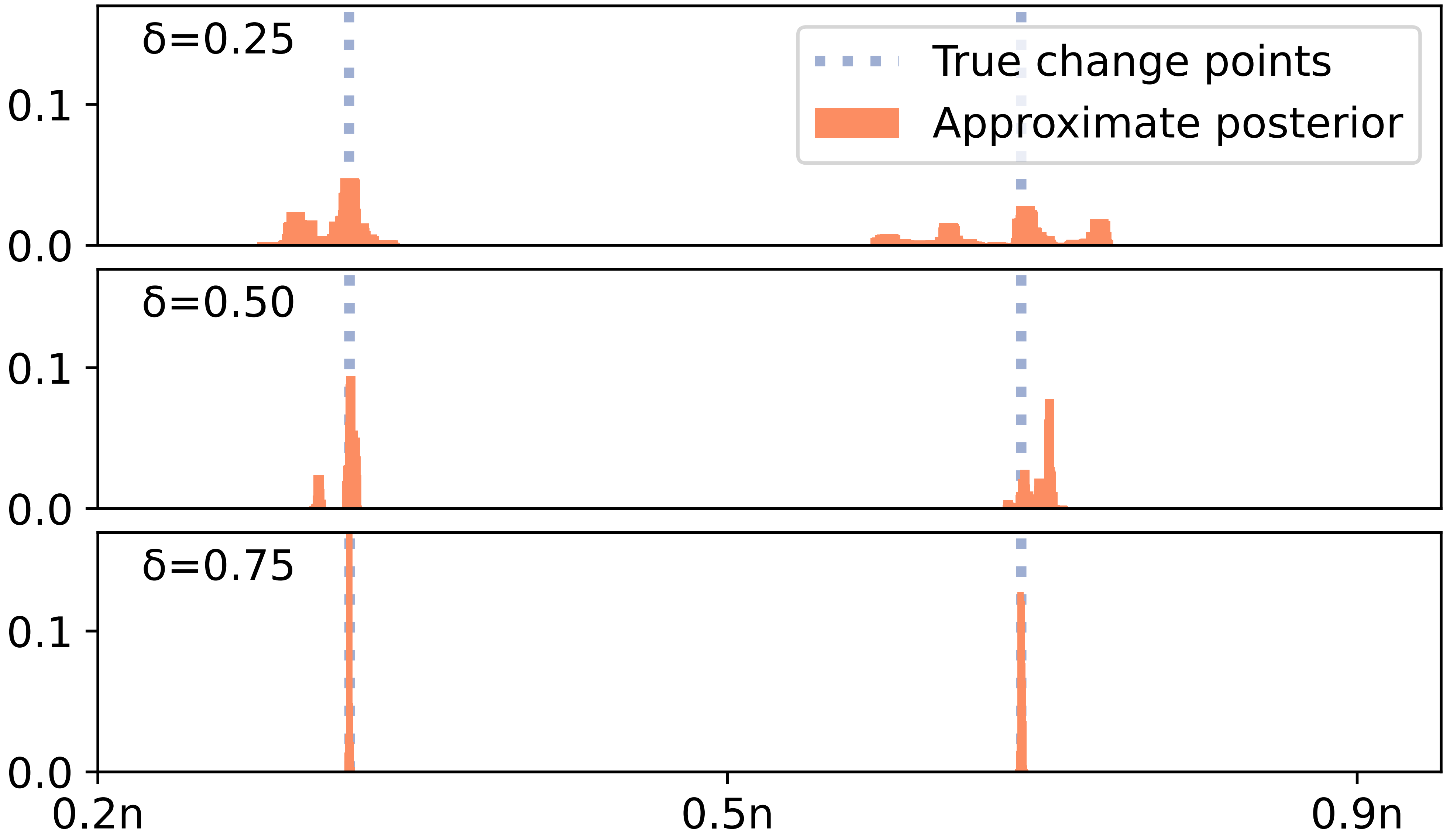

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First work to apply AMP algorithms to change point inference:

$(n, p \to \infty,\ n/p \to \delta)$

- Bayesian approach to choosing denoisers g^t, f^t
- Near-optimal computational complexity: $O(np)$
- Exact asymptotic performance guarantees (e.g., Hausdorff distance, posterior)

Future work:

generalised linear models,

beyond iid Gaussian $\boldsymbol{X}$ matrices,

online change point inference…